The opinion on Thesis entitled:
"Parallelized space-time finite element method for reducing the effects of impact loads"
presented by Mr **Dai Zhao**

to obtain the title of *Doctor in "Technical Informatics and Telecommunication"*

Reviewer: Tomasz Łodygowski, Professor at Poznan University of Technology, Poznań, Poland

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1. Formalities

The basis of this review is the request from the Secretary of the Scientific Council of the Institute of Fundamental Technological Research, Professor Zbigniew Ranachowski, DSc, PhD, Eng., to review the doctoral dissertation of Mr. Dai Zhao, MSc, supervised by Associate Professor Bartłomiej Dyniewicz, Prof. IPPT. Along with the request, I received a printed copy of Mr. Dai Zhao's doctoral dissertation.

2. Introductory remarks

The presented manuscript of PhD thesis is written in English, consists of seven main chapters and incudes Conclusions, the list of References cited in the text of the document (108 entries) and the List of symbols, abbreviations and acronyms, List of Tables and List of Figures; altogether it contains 136 pages.

In his a brief introductory remarks Mr. Dai Zhao presents the main motivations and goals of the thesis. He focuses on designig and implement the software optimized for rapid, massively parallel computing. The Author describes the contents of the thesis and some information about the adopted methodology and original contributions.

Let me stress that Mr. Dai Zhao's research was supervised by Prof. Bartłomiej Danilewicz and will be presented before the Scientific Committee of the Institute of Fundamental Technological Research.

The Author defined the main goal of the thesis in the following way:

"*The space-time finite element approach, utilizing simplex-shaped elements, allows highly efficient, massively parallelized linear and nonlinear dynamic computations that surpass the efficiency of traditional computational parallelization techniques*".

3. Contents of the thesis

In Chapter 2, the Author concluded a comprehensive literature review, which served as the point of his study. This whole thesis was divided into two fundamental parts, reflecting the main focus of Author's thesis: (1) parallel computing in structural dynamics, and (2) space-time finite element method. The cited references cover both fields of interests. As is well known, powerful computational resources, in particular HPC systems and graphical processing units (GPU), can substantially accelerate research and applications in structural dynamics. The other part of the cited literature summarizes works related to the formulation of the space-time FEM, previously proposed by Oden, Gurtin, Herrera, Kączkowski and others. It serves as a starting point for partitioning computational tasks among processors, which enables parallelization of computations in the future.

Chapter 3 focuses on the formulation of simplex-shaped space-time finite element method (STFEM). The Author's attention is focused on dynamic problems described by a hyperbolic set of differential equations. Typically, in FE technology, two integration methods are used: implicit and explicit. These two methods differ in their approach to handling the inertia matrix. The structure of this matrix has the significantly influences how the information flows through the space-time mesh during calculations. The Author discussed in detail how the information flows during the computations when using either a full or diagonal inertia matrix within a simplex-shaped STFEM mesh. The proposed formulation of simplex space-time approximation leads to one of the most important properties - the decoupling of the system of algebraic equations. This feature is fundamentally important for the parallelization of computations.

In Chapter 4, the Author examines selected engineering problems. The first example concerns one dimensional rod under axial loading within the range of small deformations, the second example involves a two-dimensional plane stress state for small deformation later extended to geometrically nonlinear problem. The Author carefully derives characteristic matrices for both cases, and for 1-D case he compared the results with an independent solution obtained using the classical FEM. The results of both computations demonstrate strong correlation, thus proving the validity of Author's concepts. When discussing large deformations of the plane-stress problem, the Candidate assumes simplex-shape functions and diligently derives all matrices relevant for further computations.

The most important part of Chapter 5 concerns the parallel computing algorithm, which is explained through a two-dimensional example. The strength of the parallel computation lies in separating tasks into several computational threads. Naturally, the effectiveness of the algorithm depends on the data transmission between the processor cores, cache, and main memory. The Author developed auxiliary tables that facilitate and simplify the steps of algorithmization. One of them (IM) describes the nonzero frames of A, B, c and D in memory. In the proposed formulation, only B and D matrices require memory reservation, and only for nonzero coefficients. Another vector (IW) partitions the IM string into sections assigned to subsequent nodes. Finally, the Author proposed the algorithm for partitioning the solution process, demonstrating a partition into eight packages. Using the proposed method for storing the coefficient matrix, operational memory as a workspace is not required. Finally, it was proven that computational performance over time scales linearly with the number of processor cores.

In Chapter 6, the Author applied the space-time FEM to two examples focusing on advanced impact protection materials. The first example involves a smart elastic material which modifies its properties according to the propagation of mechanical waves. To solve the system of governing equations, the Author used both semi-analytical solution as well as space-time FE model. A comparison of the two showed similarities in the results, confirming the effectiveness of the numerical methodology. The second, more complex example, focuses on the analysis of viscoplastic material using space-time FEM. The real advantage of using space-time FEM and parallel computing become apparent as the number of degrees of freedom increases. The Author demonstrated that CPU time increases linearly with number of nodes, while GPU time remains almost constant.

In the final two chapters, the Author briefly summarizes the contents of the thesis and formulates final conclusions, as well as possible directions for the future research, such as extension to nonlinear problems, optimization of parallel algorithms, or application to multiphysics problems.

The list of References cited in the thesis concludes the reviewed volume.

4. Remarks concerning the main achievements and the discussion

There are several points that should be emphasized as novel contributions elaborated in the thesis.

a) The Author conducted a comprehensive study on the development of space-time finite element method and applied it to solve differential equations describing the simulation of dynamic structures.
b) He introduced a methodology and a novel approach to parallel computations – the decoupling of system of equations, making it efficient for parallel processing units,
c) To prove the efficiency and accuracy, the Candidate presented a set of examples (1-D and 2-D), comparing the results with semi-analytical solutions as well as classical FEM formulations.
d) For 2-D problems, the performance verification was also discussed, comparing the solutions obtained using CPUs and GPUs, demonstrating significant advancements in computation techniques for structural engineering and dynamic analysis.

In general, I found the manuscript valuable, interesting and well-formulated, and I admire the complexity of the study.

The thesis is well-structured, logically constructed, and written in clear and precise language. However, the Author sometimes uses idiomatic expressions and overly sophisticated literary language, e.g., "*meticulously scrutinized*" – "*analyzed in* *details"*. I would also suggest distinguishing between adoption and adaptation. Other minor imprecisions do not diminish the high quality of the presentation.

After reading the manuscript, I wonder to what extend the achieved improvements in computational speed result from the mathematical formulations and decoupling the system of equations, as opposed to the use of advanced hardware architecture (GPU).

5. Conclusion

The work is a comprehensive study on a scientific problem that is both novel and important from the application perspective. The Candidate has proved his high competences in formulating and solving the scientific problems in both numerical methods and modern computational facility.

The quality of the presented thesis convinces me that Mr. Dai Zhao is a strong candidate for a Doctor of Science in the field of Technical Informatics and Telecommunications, awarded by the Scientific Council of the Institute of Fundamental Technological Research. It is my pleasure to recommend the work for oral presentation and open scientific discussion.

Poznań, February 24th, 2025

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